Jeffery D. Densmore

A Multigrid Method for the Self-Adjoint Angular Flux Form of the Radiation-Transport Equation Based on Cellwise Block Jacobi Iteration

Bettis Atomic Power Laboratory
P O Box 79
West Mifflin
PA 15122
jeffery.densmore@unnpp.gov
Daniel F. Gill
Justin M. Pounders

Cellwise block Jacobi iteration is a technique for radiation-transport calculations in which the angular flux for all directions is solved for simultaneously within a spatial cell with the angular flux in neighboring cells held fixed. Each step of the iteration then involves the inversion of a small to moderate-sized matrix for every cell. The resulting arithmetic intensity may make cellwise block Jacobi iteration suitable for advanced, heterogeneous computing architectures. However, the convergence properties of cellwise block Jacobi iteration are much different than those of other iterative schemes for radiation-transport calculations. Specifically, while convergence is rapid when spatial cells are optically thick regardless of the amount of scattering present, convergence is very slow when spatial cells are optically thin even for highly absorbing media.

To address the poor convergence properties of cellwise block Jacobi iteration for optically thin cells, we instead develop a multigrid method based on this type of iteration. If a sufficient number of spatial grids are employed in this approach, then cells on the coarsest grids should be optically thick enough that cellwise block Jacobi iteration converges rapidly. We specifically consider the Self-Adjoint Angular Flux (SAAF) form of the radiation-transport equation. For smoothing, we employ a multistage version of cellwise block Jacobi iteration. Due to the nature of the SAAF equation, we are able to determine damping parameters for each stage that are optimal in the limit of vanishingly small spatial cells, for which cellwise block Jacobi iteration without damping is ineffective as a smoother. In addition, the SAAF equation includes a term representing radiation streaming that is hyperbolic in character, and thus Direct Coarse-Grid Approximation (DCA) is inappropriate for discretization on coarse grids. We instead employ a hybrid approximation formed by applying Galerkin coarse-grid approximation based on full-weighting restriction and bilinear interpolation to the streaming term and DCA to the remaining terms. With a set of numerical examples, we demonstrate that our multigrid method yields robust convergence regardless of cell optical thickness.