William W. Dai

On Operator-Splitting Approach for Plasma 3-T Radiation Diffusion in Two and Three Dimensions

Los Alamos National Laboratory
Mail Stop D413
Los Alamos
NM 87545
dai@lanl.gov
William W. Dai
Anthony J. Scannapieco

Solving plasma 3-T radiation diffusion equations is a critical step in multi-physics simulations, Often these simulations involve mixed cells, and within a mixed cell there are more than one material, no matter how fine the resolution of a simulation is. Treatment of mixed cells is important for many applications. Since materials within a mixed cell could be in a state of thermally non-equilibrium, and material property of mixture of materials is unknown, a mixed cell is decomposed to a set of sub-cells through material interface reconstruction so that each sub-cell contains only one material. The sub cells thus generated are general polygons in two-dimensions and polyhedrons in three dimensions. The plasma 3-T radiation diffusion equations are solved on the mesh with these general polygons and polyhedrons.

Two typical implicit methods are the backward Euler method and Crank-Nicolson method. The backward Euler method is first order accurate in time, but numerical errors in the method undergo quick damping for large time steps. Therefore the method is very useful for large time steps and steady states. Although Crank-Nicolson method is second order accurate, numerical errors do not damp out for large time steps, and significant numerical errors will be introduced when time steps are large. The formulation for large time steps is very important for problems involving dramatically different materials even for time-dependent problems. A given time step may be considered so large for some material that the formulation for steady state is more appropriate for the material than the formulation with the second order of accuracy, but the time step is so small for other materials that accuracy in time is more important. The scheme we will present is second order accurate in space and time, works for any size of time steps, and will give exact steady states when time steps are very large.

For systems of multi-materials with dramatically different material properties, the correct treatment for the discontinuity of material properties is important. A typical approach for this is to use mathematical approximations, which could introduce numerical errors when thermal properties of two materials are very different. We use the governing physics principle to give formula for effective diffusion coefficient across a material interface for flux calculation on polyhedral meshes.

In addition to the aspects mentioned above, we will focus on another issue in numerical simulations for plasma 3-T radiation equations, i.e., numerical treatment for interaction between radiation and material that involves nonlinearity. The 3-T radiation diffusion equations are often solved through some operator-splitting technique. Several approaches that are typically used in numerical simulations have serious drawbacks for some problems.

The numerical scheme to be presented is intended for a code of multi-physics simulations. It is an extension of previous work to include mixed cells, unstructured meshes, fully coupling, full nonlinearity, and operator-splitting. For mixed cells, we consider the interface reconstruction with any number of materials in both two- and three-dimensional meshes with adaptive mesh refinement (AMR). In this talk, we will first describe the procedure for interface reconstruction for both two- and three-dimensions for AMR meshes with any number of materials, and then we will present the scheme for the set of equations for 3-T radiation diffusion in both two- and three-dimensions. We will demonstrate serious errors of typical operator-splitting approaches for some problems, and present our operator-splitting techniques.

One operator-splitting approach is to decompose plasma 3-T radiation diffusion equations into two steps, a set of diffusion equations without interaction among radiation, electrons and ions, and a set of equations for the interactions without diffusion. In the first step, the set of diffusion equations are fully implicitly solved. In the second step, the coupling among radiation, electrons and ions is also fully implicitly solved. Through the first step of the operator-splitting approach, radiation temperature diffuses across the material interface, while temperatures of electrons and ions may diffuse little. During the second step of the operator-splitting, almost all the increase of radiation energy of the heated material transfers to local electrons and ions, but electrons and ions increase their temperatures very little because of their large material heat capacity. As a result, three temperatures barely diffuse through this operator-splitting approach.

A slightly change of this operator-splitting is an approach with three steps, a half time step of coupling, one time step of diffusion, and a half time step of coupling. Unfortunately, this operator-splitting approach will get a similar result.

Another operator-splitting approach for 3-T radiation diffusion equations is more sophisticated than the one described above, but bears another serious problem. This approach decompose the set of basic equations into four steps, each of step is fully implicitly solved. But, due to an inappropriate treatment for the nonlinear terms, electrons and ions are much over-heated.

In additional to a fully nonlinear scheme in which material and radiation are fully coupled, we will present a scheme in which the nonlinearity is properly treated, and an operator-splitting technique is used, which generates reasonable solutions for those problems for which the previous operator-splitting approaches fail.