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Model Reduction for Uncertainty Quantification of Large-scale Systems

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Uncertainty quantification is becoming recognized as an essential aspect of development and use of numerical simulation tools, yet it remains computationally intractable for large-scale complex systems characterized by high-dimensional uncertainty spaces. For example, efficient numerical tools to support decision under uncertainty are essential in many settings. Examples include the design, optimization and control of complex engineered systems, and the real-time characterization of and response to hazardous events. In both these settings, it is essential to address the many different sources and types of uncertainty, including those stemming from the fidelity of the underlying mathematical models. However, solving the resulting stochastic optimization problem may be computationally infeasible, especially when the underlying simulation models result from discretization of systems of partial differential equations. Model reduction thus has an important role to play in producing low-order approximate models that retain the essential system dynamics but that are fast to solve.

Reduced-order models are commonly derived using a projection framework in which the governing equations of the forward model are projected onto a subspace of reduced dimension. This reduced subspace is defined via a set of basis vectors. Methods to compute the basis in the large-scale setting include approximate balanced truncation, Krylov-subspace methods, proper orthogonal decomposition (POD), and reduced basis methods. Balanced truncation and Krylov-subspace methods are in general restricted to linear systems. POD and reduced basis methods use empirical information generated from sampled solutions ("snapshots") of the large-scale system to create the reduced basis.

This talk will discuss formulation and derivation of projection-based reduced-order models for uncertainty quantification of systems governed by partial differential equations. In particular, we demonstrate the use of reduced models for uncertainty propagation, solution of statistical inverse problems, and optimization under uncertainty. Our methods use state approximations through the POD, reductions in parameter dimensionality through parameter basis approximations, and the discrete empirical interpolation method for efficient evaluation of nonlinear terms.