Chad Westphal

FOSLL* For Nonlinear Partial Differential Equations

1090 Milo Cir A
Lafayette
CO 80026
westphac@wabash.edu
Eunjung Lee
Thomas Manteuffel

Least-squares finite element methods are designed around the idea of minimizing discretization error in an appropriate norm. For sufficiently regular, elliptic-like problems, a first-order system least squares (FOSLS) approach may be continuous and coercive in the H^1 norm, yielding solutions accurate in H^1. For problems posed in high aspect ratio domains (e.g., flow through a long channel), approximations on coarse resolutions may have error whose L^2 norm is large relative to the H^1 seminorm. The first order system LL^* (FOSLL*) approach seeks to minimize the residual of the equations in a dual norm induced by the differential operator, yielding a better L^2 approximation. Mass conservation in fluid flow, for example, is greatly enhanced by such an approach. In this talk, we extend this general framework to nonlinear problems.

Newton's method is a typical outer iteration for an efficient finite element approximation of nonlinear partial differential equations. We present the framework for an inexact Newton iteration based on a FOSLL* approximation to each linearization step and establish theory for convergence. Numerical results are presented for a velocity-vorticity-pressure formulation of the steady incompressible Navier-Stokes equations, and we discuss extensions and comparisons to the more typical Newton-FOSLS approach.