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A block-diagonal algebraic multigrid preconditioner for the Brinkman problem

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The Brinkman model is a unified law governing the flow of a viscous fluid in cavity (Stokes equations) and in porous media (Darcy equations). It was initially proposed as a homogenization technique for the Navier-Stokes equations. Typical applications of this model are in underground water hydrology, petroleum industry, automotive industry, biomedical engineering, and heat pipes modeling. In this talk, we present a novel mixed formulation of the Brinkman problem. Introducing the flow's vorticity as additional unknown, this formulation leads to a uniformly stable and conforming discretization by standard finite element (Nédélec, Raviart-Thomas, piecewise discontinuous). Based on stability analysis of the problem in the $H(\text{curl}) - H(\text{div}) - L^2$ norms, we derive a scalable block diagonal preconditioner which is optimal in the constant coefficient case. Such preconditioner is based on the auxiliary space AMG solvers for $H(\text{curl})$ and $H(\text{div})$ problems available in hypre (http://www.llnl.gov/CASC/hypre/). The theoretical results are illustrated by numerical experiments.

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