
Ray Tuminaro
**A Galerkin-based Methodology for Constructing AMG
Grid Transfers**

MS 9159
PO Box 969
Livermore
CA 94551
`rstumin@sandia.gov`
Michael Gee
Tobias Wiesner

A methodology is proposed for constructing algebraic multigrid (AMG) transfer operators. A key feature of the approach is generality as it can be applied to nonsymmetric systems, can emphasize accuracy for different user-defined modes, and can incorporate a variety of coarsening strategies and grid transfer sparsity patterns. The basic concept centers on approximation of idealized Schur complement-based grid transfers via a Galerkin projection. To explain the idea, consider the solution of the matrix equations

$$Au = b \tag{1}$$

via an algebraic multigrid algorithm. Let P and R represent prolongation and restriction matrices respectively and now examine the equations

$$AP = 0 \quad \text{and} \quad RA = 0. \tag{2}$$

Obviously, only trivial solutions are obtained if A is square and nonsingular and no restrictions are placed on P and R . As in classical AMG, let the vertices be partitioned into two sets. The c -point set corresponds to vertices that remain on the next coarsest level while all other vertices are in the f -point set. These sets induce a partitioning of P and R :

$$R = \begin{pmatrix} R_f & R_c \end{pmatrix} \quad \text{and} \quad P = \begin{pmatrix} P_f \\ P_c \end{pmatrix}. \tag{3}$$

It is easy to show that idealized Schur complement-based grid transfers are obtained if we restrict the solution spaces so that $P_c = I$ and $R_c = I$ and if we enforce zero residuals only at f -points. That is, $(AP)_f = 0$ and $(RA)_f = 0$. More generally, this can be formulated as a Galerkin process where spaces for solutions and residuals define a projection of (2). While Schur complements are generally not computationally attractive, we investigate other test and trial spaces for residuals and solutions. In particular, spaces are discussed which limit the sparsity pattern of the grid transfers while ensuring that a few modes

(e.g. constants) are accurately transferred. We then illustrate that these spaces lead to practical and efficient AMG grid transfer operators which can be computed by a few iterations of a Krylov process. The final algorithm is closely related to energy minimizing algebraic multigrid ideas (where the minimization properties of the Krylov method implicitly define minimization properties and norms associated with the grid transfer basis functions), but it is also suitable for nonsymmetric systems.

Numerical results are given to demonstrate flexibility/adaptability and to highlight how this mathematical flexibility leads to some interesting possibilities for AMG software. Examples are taken from applications in fluid flow, semiconductor modeling, and ice sheet fracture. These include cases where grid transfers with irregular sparsity patterns are needed as well as some challenging nonsymmetric linear systems.