Ralph Thesen
Efficent adaptive solution of multilevel-systems

Fraunhofer Institute for Algorithms and Scientific Computing SCAI
Schloss Birlinghoven
D-53754 Sankt Augustin
Germany
`ralph.thesen@scai.fraunhofer.de`

The Gauss-Southwell-algorithm is the greedy grandfather of the well known Gauss-Seidel-algorithm. This adaptive algorithm solves in every step the equation with the largest current residual instead of a priori fixed running order. We present a method to find the maximum residual with minimum costs. We employ a heap data structure to store the residual which allows us not only to find the maximum residual very fast, but also to update entries in an incremental manner.

The greedy choice of the largest residual leads to a fast convergence, since the iterative solver adapts itself to the structure of the linear system. The beneficial effects from utilizing the structural information in a greedy manner dominate the extra costs in a total balance. In particular multilevel systems, which are highly structured, are well suited for such an adaptive algorithm.

This adaptive strategy shows a dramatic speedup for anisotropic problems compared with the non-adaptive methods. Convergence rates are shown for a class of highly anisotropic linear elliptic problems. In all examples our proposed method is substantially faster than the Gauss-Seidel-algorithm.