Jyoti Saraswat Multigrid solution of a distributed optimal control problem constrained by a semilinear elliptic PDE

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In this work we discuss the approximation properties of multigrid preconditioners for the linear systems arising in the solution process of a distributed optimal control problem constrained by a semilinear elliptic PDE. After discretizing the PDE using standard finite elements, we solve the resulting discrete optimal control problem via Newton's method. As is standard for large-scale problems, adjoints are used for gradient computations and for Hessian-vector multiplications, and the linear systems are solved using matrix-free conjugate gradient preconditioned by multigrid preconditioners related to the ones introduced by Draganescu and Dupont (2008) for linear ill-posed problems. The main contribution of this work is to show that, as in the case of optimal control of linear elliptic PDEs, the multigrid preconditioners approximate the Hessian to optimal order with respect to the mesh-size.