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Geoffrey Sanders  
**Eigenvectors of Matrices Associated with Scale-Free  
Graphs**

Lawrence Livermore National Laboratory  
Box 808  
L-561  
Livermore  
CA 94551-0808  
`sanders29@llnl.gov`  
Van Emden Henson  
Panayot Vassilevski

A scale-free graph is characterized as having vertices with a degree distribution that follows a power law. Clustering, ranking, and measuring similarity of vertices on scale-free graphs are all quantities that network scientists often calculate. We describe a few network calculations that are obtained by solving eigensystems or linear systems involving matrices whose sparsity structure is related to an underlying scale-free graph. For many large systems of interest, classical iterative solvers (such as conjugate gradient and lanczos) converge with prohibitively slow rates, due to ill-conditioned matrices and small spectral gap ratios. Scalable preconditioners for this class of problem are of considerable research interest. We consider constructing algebraic multigrid (AMG) preconditioners, which make use of coarse approximation of eigenspaces to accelerate convergence. We discuss several interesting properties the eigenvectors of scale-free graphs have that are not present in mesh-like graphs. For example, these graphs have a large number of locally supported eigenvectors (LSEVs), eigenvectors that are each nonzero only in a small, connected region. We discuss how such properties can be used to coarsen graphs with potentially high spectral accuracy and ultimately improve the scalability of preconditioned iterative methods.