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Block Preconditioners for an Exact Penalty Viscoresistive MHD Formulation

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The magnetohydrodynamics equations are used to model the flow of electrically conducting fluids in such applications as liquid metals and plasmas. The equations are a system of non-self adjoint, non-linear PDEs which couple the Navier-Stokes equations for fluids and Maxwells equations for electromagnetics. They can span over a range of length- and time-scales, necessitating robust, accurate approximation techniques. There has been recent interest in fully coupled solvers for the MHD system because they allow for fast steady-state solutions that do not require pseudo-time stepping. When the fully coupled system is discretized, the strong coupling can make the resulting algebraic systems difficult to solve, requiring effective preconditioning of iterative methods for efficiency.

In this work, we consider a finite element discretization of an exact penalty formulation for the stationary MHD equations (as proposed by Gerbeau, 2000). This formulation has the benefit of implicitly enforcing the divergence free condition on the magnetic field without requiring a Lagrange multiplier. We consider extending block preconditioning techniques developed for the Navier-Stokes equations (such as those of Elman et al., 2008) to the full MHD system. We analyze operators arising in block decompositions from a continuous perspective and apply approximate commutation arguments in order to develop new preconditioners that account for the multi-physics coupling. We demonstrate the quality of these preconditioners over a range of parameters on a variety of two-dimensional test problems.