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**Algebraic Multigrid (AMG) for Saddle Point Systems - A  
self-stabilizing approach**

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We present an Algebraic Multigrid (AMG) approach to the solution of Stokes-type saddle point problems of the form

$$\mathcal{K} : \begin{pmatrix} \mathcal{V} \\ \mathcal{W} \end{pmatrix} \rightarrow \begin{pmatrix} \mathcal{V} \\ \mathcal{W} \end{pmatrix}, \quad \mathcal{K} = \begin{pmatrix} A & B^T \\ B & -C \end{pmatrix}$$

where  $A$  is positive definite and  $C$  is positive semi-definite.

In particular, we address the two main challenges to the construction of a saddle point AMG method,

1. The indefinite matrix  $\mathcal{K}$  does not define an inner product and norm, hence, it is not possible to carry out the AMG convergence proof in terms of the energy norm (as is done in AMG for symmetric positive matrices),
2. The coarse grid matrix computed by the Galerkin product may not be invertible at all.

The starting point for the construction of our AMG method for saddle point systems is the inexact symmetric Uzawa smoothing scheme introduced in [1]. We set up the coarse grids and prolongation operators for the spaces  $\mathcal{V}$  and  $\mathcal{W}$  independently. From these per-space interpolation operators, we assemble the global interpolation and restriction operators  $\mathcal{P}$  and  $\mathcal{R}$  such that an inf-sup condition for  $K$  implies an inf-sup-condition for the coarse system computed by the Galerkin product

$$\mathcal{K}^C = \mathcal{R}\mathcal{K}\mathcal{P}.$$

Especially, our approach does not require that the coarse grids for  $\mathcal{V}$  and  $\mathcal{W}$  are adapted to each other. Instead, the stability and thus the invertibility of the coarse grid operator is ensured by the construction of the transfer operators. We also give a two-grid convergence proof for our method based on the framework of [2], which does not depend on the existence of an energy norm.

# Bibliography

- [1] J. SCHÖBERL AND W. ZULEHNER. On Schwarz-type Smoothers for Saddle Point Problems. *Numerische Mathematik* 95 (2003) 377-399.
- [2] Y. NOTAY. Algebraic analysis of two-grid methods: The nonsymmetric case. *Numer. Linear Algebra Appl.* 17 (2009) 73-96.