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Hybrid First-order System Least Squares Finite Element Methods With Application to Stokes Equations

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In this talk, we combine the FOSLS method with the FOSLL* method to create a Hybrid method. The FOSLS approach minimizes the error, $\mathbf{e}^h = \mathbf{u}^h - \mathbf{u}$, over a finite element subspace, $\mathcal{V}^h$, in the operator norm, $\min_{\mathbf{u}^h \in \mathcal{V}^h} \|L(\mathbf{u}^h - \mathbf{u})\|$. The FOSLL* method looks for an approximation in the range of L^*, setting $\mathbf{u}^h = L^*\mathbf{w}^h$ and choosing $\mathbf{w}^h \in \mathcal{W}^h$, a standard finite element space. FOSLL* minimizes the $\mathbf{L}^2$ norm of the error over $L^*(\mathcal{W}^h)$, that is, $\min_{\mathbf{w}^h \in \mathcal{W}^h} \|L^*\mathbf{w}^h - \mathbf{u}\|$. FOSLS enjoys a locally sharp, globally reliable, and easily computable a posterior error estimate, while FOSLL* does not.

The hybrid method attempts to retain the best properties of both FOSLS and FOSLL*. This is accomplished by combining the FOSLS functional, the FOSLL* functional, and an intermediate term that draws them together. The Hybrid method produces an approximation, $\mathbf{u}^h$, that is nearly the optimal over $\mathcal{V}^h$ in the graph norm, $\|\mathbf{e}^h\|^2_{\mathcal{G}} := \frac{1}{2}\|\mathbf{e}^h\|^2 + \|L\mathbf{e}^h\|^2$. The FOSLS and intermediate terms in the Hybrid functional provide a very effective a posteriori error measure.

We show that the hybrid functional is coercive and continuous in the graph-like norm with modest constants, $c_0 = 1/3$ and $c_1 = 3$; that both $\|\mathbf{e}^h\|$ and $\|L\mathbf{e}^h\|$ converge with rates based on standard interpolation bounds; and that if LL^* has full H^2 regularity, the $\mathbf{L}^2$ error, $\|\mathbf{e}^h\|$, converges with a full power of the discretization parameter, h, faster than the functional norm. Letting $\tilde{\mathbf{u}}^h$ denote the optimum over $\mathcal{V}^h$ in the graph norm, we also show that if superposition is used, then $\|\mathbf{u}^h - \tilde{\mathbf{u}}^h\|_{\mathcal{G}}$ converges two powers of h faster than the functional norm. Numerical tests are provided to confirm the efficiency of the Hybrid method.