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Hybrid First-order System Least Squares Finite Element Methods With Application to Stokes Equations

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In this talk, we combine the FOSLS method with the FOSLL* method to create a Hybrid method. The FOSLS approach minimizes the error, $^h = ^h - $, over a finite element subspace, ^{h}, in the operator norm, $\min_{^h \in ^h} \|L(^h -)\|$. The FOSLL* method looks for an approximation in the range of L^*, setting $^h = L^{*h}$ and choosing $^h \in ^h$, a standard finite element space. FOSLL* minimizes the ^{2} norm of the error over $L^*(^h)$, that is, $\min_{^h \in ^h} \|L^{*h} - \|$. FOSLS enjoys a locally sharp, globally reliable, and easily computable a posterior error estimate, while FOSLL* does not.

The hybrid method attempts to retain the best properties of both FOSLS and FOSLL*. This is accomplished by combining the FOSLS functional, the FOSLL* functional, and an intermediate term that draws them together. The Hybrid method produces an approximation, ^{h}, that is nearly the optimal over ^{h} in the graph norm, $\|^h\|^2 := \frac{1}{2}\|^h\|^2 + \|L^h\|^2$. The FOSLS and intermediate terms in the Hybrid functional provide a very effective a posteriori error measure.

We show that the hybrid functional is coercive and continuous in the graph-like norm with modest constants, $c_0 = 1/3$ and $c_1 = 3$; that both $\|^h\|$ and $\|L^h\|$ converge with rates based on standard interpolation bounds; and that if LL^* has full H^2 regularity, the ^{2} error, $\|^h\|$, converges with a full power of the discretization parameter, h, faster than the functional norm. Letting $^{\neg h}$ denote the optimum over ^{h} in the graph norm, we also show that if superposition is used, then $\|^h - ^{\neg h}\|$ converges two powers of h faster than the functional norm. Numerical tests are provided to confirm the efficiency of the Hybrid method.