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**Finite element solution of a Schelkunoff complex vector  
potential for frequency domain, EM 64257;eld simulation**

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A novel method for the 3-D diffusive electromagnetic (EM) forward problem is developed and tested. A Lorentz-gauge, Schelkunoff complex vector potential is used to represent the EM field in the frequency domain and the nodal finite element method is used for numerical simulation. The potential allows for three degrees of freedom per node, instead of four if Coulomb-gauge vector and scalar potentials are used. Unlike the finite-difference method, which minimizes error at discrete points, the finite element method minimizes error over the entire domain cell volumes and may easily adapt to complex topography. Existence and uniqueness of this continuous Schelkunoff potential is proven, boundary conditions are found and a governing equation satisfied by the potential in weak form is obtained. This approach for using a Schelkunoff potential in the finite element method differs from others found in the literature. If the standard weak form of the Helmholtz equation is used, the obtained solution is continuous and has continuous normal derivative across boundaries of regions with different physical properties; however, continuous Schelkunoff potential components do not have continuous normal derivative, divergence of the potential divided by (complex) conductivity and magnetic permeability is continuous instead. Two weak forms of the governing equation are tried. Both of them produce a system matrix that is ill-conditioned and as a result iterative schemes do not converge. A different idea is tried next, instead of representing electric field by a Schelkunoff potential, magnetic field is represented in a similar way. When it is assumed that magnetic permeability  $\mu$  is constant, a convenient weak form of the governing equation is obtained. This form is tested numerically on a simple model of a conducting prism in a resistive whole space and the code gives a similar results to an independent finite difference code.