Tzanio Kolev

Parallel Auxiliary Space AMG Solver for H(Div) Problems

Center for Applied Scientific Computing
Lawrence Livermore National Laboratory
P O Box 808
L-560
Livermore
CA 94551
tzanio@llnl.gov
Panayot Vassilevski

In this talk we present a family of scalable preconditioners for matrices arising in the discretization of $\boldsymbol{H}(\mathrm{div})$ problems using the lowest order Raviart-Thomas finite elements. Our approach belongs to the class of "auxiliary space"-based methods and requires only the finite element stiffness matrix plus some minimal additional discretization information about the topology and orientation of the mesh entities. We outline the theory which is based on certain regular decompositions for $\boldsymbol{H}(\mathrm{div})$ spaces by relating them to a previously studied "inf-sup" condition for parameter-dependent Stokes problems. We provide a detailed algebraic description of the method, its parallel implementation and different variants of this parallel auxiliary space divergence solver. Also, we discuss its relations to the Hiptmair-Xu auxiliary space decomposition of Raviart-Thomas spaces [1], the related approaches [4, 5], as well as the auxiliary space Maxwell solver (AMS) [2] from the *hypre* library [3]. An extensive set of numerical experiments demonstrate the robustness and scalability of our implementation on large-scale parallel $\boldsymbol{H}(\mathrm{div})$ problems with large jumps in the material coefficients.

Bibliography

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[3] *hypre*: a library of high performance preconditioners, *http://www.llnl.gov/CASC/hypre/*

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