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On preconditioners for generalized sparse grid generating systems

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Tensor products of one-dimensional multilevel systems can be used to represent multivariate functions. Then, by a proper truncation of the resulting series expansion, we can construct problem-dependent sparse grids, which allow us to efficiently approximate higher-dimensional problems for various norms and smoothness classes.

We discuss additive Schwarz preconditioners for the corresponding systems of linear equations. The problem of finding the optimal diagonal scaling for the generating system subspaces can be solved by means of Linear Programming. For e.g. H^1-elliptic problems, an optimally scaled regular sparse grid generating system exhibits a condition number of the order $O(k^{d-2})$ for level k in d dimensions.

This is suboptimal compared to the $O(1)$ condition numbers realized by prewavelet discretizations that directly rely on multiresolution norm equivalences. However, we will discuss an approach that likewise realizes $O(1)$ condition numbers in the generating system without specifically discretising the detail spaces via more complicated basis functions.

This is joint work with M. Griebel (University of Bonn) and P. Oswald (Jacobs University Bremen).