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Development and Parallel Implementation of Algebraic Multigrid Algorithms Suited for eXtended Finite Element Methods to Model Three-Dimensional Crack Problems.

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Algebraic Multigrid (AMG) does not work very well when directly applied to fracture mechanics problems modeled with extended finite element methods (XFEM) because of a number of factors. For instance, since the XFEM enrichment functions have a local support near the crack at original fine mesh, they cannot be directly or accurately represented on the coarser discretizations generated by AMG. The value of XFEM for modeling fracture mechanics problems lies in its ability to address discontinuities by directly incorporating them in the shape functions. These shape functions span seamlessly across the elements containing the discontinuities, so that the mesh itself need not conform to them. However, this results in graph patterns of the tangent stiffness matrix that contain strong couplings that cross crack discontinuities. Moreover the different number of degrees of freedom in XFEM for regular mesh nodes and enriched nodes poses a difficulty for AMG.

In the authors' earlier works [1-3], AMG methods were adapted to work for XFEM and applied to solve two dimensional crack problems. In [1], the Schur complement of the XFEM linear system (in which the enriched degrees of freedom were condensed out) was used to develop a Hybrid-AMG method such that crack-conforming aggregates were formed. Domain devomposition methods were explored in [3] such that AMG worked only on the "healthy" subdomains that do not contain discontinuities thereby retaining its convergence optimality. An inexact Schwarz method was developed to combine the resulting solution for the healthy subdomain with that obtained from applying a direct-solve operation on the "cracked" subdomain. Another alternative approach was presented in [2] and this involved transforming the original XFEM linear system into a modified system that was amenable for a direct application of AMG. It was shown that if only Heaviside-enrichments are present, a simple transformation based on the phantom-node approach is readily available and which decouples the linear system along the discontinuities and retains only the regular degrees of freedom in the AMG mesh hierarchies. A back-transformation of the resulting solution vector provided the solution to the original linear system before transformation.

In this work, the last approach (transformation) has been further extended for three dimensional crack problems modeled with XFEM using user element subroutines developed in the nonlinear finite element program FEAP. These user elements have been further adapted to work with the parallel version of the program ParFEAP. The proposed AMG solution algorithms are implemented using the ML and MueLu packages on the Trilinos framework developed by Sandia National Laboratories. The solution subroutines within ParFEAP which are originally developed for PETSC have been modified to work with the Trilinos framework. The new programs allow for the proposed XFEM and AMG methods to be scaled up for solving large problems on parallel computing clusters with distributed memory architectures. Various numerical examples will be presented to verify the accuracy of the resuting solutions and the convergence properties of the AMG algorithm. The parallel scalability performance of the implementation will also be discussed.

REFERENCES:

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