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Comparison of Continuous and Discontinuous Galerkin Finite Element Methods for Parabolic Differential Equations

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A number of different discretization techniques and algorithms have been developed for approximating the solution of parabolic partial differential equations. A standard approach, especially for applications that involve complex geometries, is the classic continuous Galerkin Finite Element Method (GFEM). This approach has a strong theoretical foundation and has been widely and successfully applied to this catagory of differential equations. One challenging catagory of problems, however, are equations that include an advective term that large relative to the second-order, diffusive term. For these advection dominated problems, the continuous GFEM discretization can become unstable and yield highly inaccurate results. An alternative to the continuous GFEM is the discontinuous GFEM, and, through the use of a numerical flux term used in deriving the weak form, the discontinuous approach has the potential to be much more stable in highly advective problems. However, the discontinuous GFEM also has significantly more degrees-of-freedom due to the replication of nodes along element edges and vertices. This paper compares the computational cost, stability, and accuracy (when possible) of continuous and discontinuous GFEM for four different test problems including the advection-diffusion equation, viscous Burgers' equation, and the Turing pattern formation equation system. The comparison is performed using as much shared code as possible between the two algorithms and direct, iterative, and multilevel linear solvers. The results show that, for implicit time stepping, the continuous GFEM is typically 5-20 times less computationally expensive than the discontinuous GFEM using the same finite element mesh and element order. However, the discontinuous GFEM is significantly more stable than the continuous GFEM for advection dominated problems and is able to obtain accurate approximate solutions for cases where the classic, unstabilized continuous GFEM fails.