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Fast iterative algorithm for shape reconstruction in image processing

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Many image processing tasks are naturally expressed as shape optimization problems, in which the main goal is to reconstruct a set of shapes, such as curves in 2d or surfaces in 3d, based on some given data. Examples of such tasks are image segmentation, the task of identifying the objects or uniform regions in a given image, or certain inverse problems, where the target is not the material medium itself, but the structures or anomalies in the medium. Often, at the outset of such problems, one has the data at hand and some rough idea of what the shapes should or should not look like, but does not have a direct way to compute the shapes from the data. In such a situation, an effective approach is to define a shape energy and to compute shapes that minimize this problem-specific shape energy. The shape energy would typically have a data fidelity component minimized by the correct shapes (based on the physics that relates the shape to the data), also a smoothness term, such a length or area penalty that favors smoother shapes over nonsmooth shapes.

In this work, we propose an efficient and robust iterative algorithm to compute the solutions of such shape reconstruction problems in image processing. At each iteration of the algorithm, we compute a gradient descent velocity and use it to deform a candidate shape in a way that keeps decreasing its energy until the algorithm stops at a minimum of the energy. The starting point of our method is a continuous shape optimization formulation and analytical computation of the first and second shape derivatives based on the energy formulation. This information enables us to devise Newton-type gradient descent methods that converge in reduced number of iterations and significantly reduce the actual computation time. Moreover we are able to impose the choice of descent velocities to be in H^1 Sobolev spaces defined on the shape and perform the gradient descent with smoother velocities. This proves to be advantageous in the case of noisy and incomplete data; it reduces the computation time and helps preserve a quality representation.

We discretize the continuous algorithm using the finite element method. Unlike

typical partial differential equation (PDE) applications, we need to discretize both the geometry (which is also an unknown) and the continuous equations (PDEs and integrals) associated with the gradient descent. As the meshes representing the shapes and regions move and deform through the iterations, we need to continually update and maintain the mesh in order to preserve accuracy. Geometric accuracy, as well as numerical accuracy, is achieved by maintaining a few errors indicators, the total of which is used to ensure that we have a valid gradient descent direction at each iteration. These error indicators, in conjunction with geometric mesh adaptivity, are used to tune the accuracy of the method at each iteration and are key to the economy of the algorithm. In particular, we employ a coarse-to-fine continuation approach and impose accuracy gradually as we get close to the solution, thus save in computation when we are far away from the optimum. Putting these ingredients together results in an effective algorithm applicable to many shape reconstruction problem in imaging. We demonstrate the power of the method with several examples.