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Spatially Varying Stochastic Expansions for Embedded Uncertainty Quantification

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Uncertainty quantification (UQ) is increasingly recognized as necessary to assess the effect of variability of input parameters on a physical model. For the solution of stochastic PDEs, existing discretizations are based on a tensor product between the deterministic basis and the stochastic basis. The implicit assumption made by the tensor product is that the required order of the stochastic approximation is fixed throughout the domain. However, solutions to many PDEs of interest exhibit spatially localized features that may result in uncertainty being severely over or under resolved by existing discretizations. The result of under resolution is a loss in accuracy, and for over resolution excess computational cost is incurred.

To address this issue, we will consider using a discretization where the order of the stochastic expansion is allowed to vary across the physical domain. This is achieved by a Galerkin projection onto a spatially varying basis. The challenge of this method is in the construction of such a basis which, due to the fine-grain level of granularity, compels us to use embedded UQ techniques that require a direct modification of the PDE code. This is in contrast to black-box techniques, like Monte-Carlo or stochastic collocation, which do not appear to be applicable to the spatially varying discretization.

In this talk, we propose an adaptive algorithm using numerical error indicators to construct the spatially varying discretization. Furthermore, we will discuss the changes to the stochastic weak form that make the spatially varying discretization possible. Finally, we will demonstrate the performance of the adaptive algorithm with several numerical examples and provide comparisons to other embedded UQ techniques.