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CS-MINRES: A Krylov Subspace Method for Complex Symmetric Linear Systems and Least-Squares Problems

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While there is no lack of high-quality Krylov subspace solvers for (complex) Hermitian systems, there are few for complex symmetric systems, which have become increasingly important in modern applications including quantum dynamics, electromagnetics, power systems, and many more.

For a large consistent complex symmetric system $Ax = b$, i.e., $A = A^T$, one may apply a non-Hermitian Krylov subspace method such as BiCG, CGS, or GMRES, overlooking the symmetry of A, or a Hermitian Krylov solver such as CG, SYMMLQ, or MINRES on the equivalent normal equation or an augmented system twice the original dimension. These have the disadvantages of increasing either memory, conditioning, or computational costs. An exception is a special version of QMR by Freund (1992), but that may be affected by non-benign breakdowns unless look-ahead is implemented; furthermore, it is designed for consistent problems only.

We describe CS-MINRES, a new algorithm for solving linear systems and least-squares problems with complex symmetric coefficient matrices. In all cases, CS-MINRES computes the unique minimum-length, i.e., pseudoinverse, solution. It may be regarded as an extension of MINRES by Paige and Saunders (1975) or the more recent MINRES-QLP by Choi, Paige and Saunders (2011). CS-MINRES does not suffer from non-benign breakdowns and is immediately applicable to real symmetric problems, in which case it is mathematically equivalent to MINRES-QLP.

Like MINRES and MINRES-QLP, CS-MINRES has elegant theoretical properties, and is sufficiently robust for ill-conditioned and ill-posed problems. We will present extensive numerical experiments to demonstrate the scalability and robustness of CS-MINRES.