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A Multilevel Preconditioner for the Bingham Fluid Flow in Mixed Variables

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The Bingham fluid flow is a Stokes-type flow with shear-dependent viscosity. If $\mathbf{Du} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ and $|\mathbf{Du}| = \sqrt{\mathrm{tr}(\mathbf{Du}^2)}$, its equations read

$$\begin{cases} -\nabla\cdot\tau + \nabla p &= \mathbf{f}, \\ -\nabla\cdot\mathbf{u} &= 0 \\ +B.C., & \end{cases}$$

and

$$\begin{cases} \tau = 2\mu\mathbf{Du} + \tau_s\frac{\mathbf{Du}}{|\mathbf{Du}|}, & \text{if } |\mathbf{Du}| \neq 0, \\ |\tau| \leq \tau_s, & \text{if } |\mathbf{Du}| = 0, \end{cases}$$

where the velocity $\mathbf{u} \in \mathbb{R}^n$, $n = 2, 3$ and $p \in \mathbb{R}$ are the unknowns and μ, τ_s are given constants. A major difficulty of solving the Bingham equations numerically is the fact that its equations are singular for $\mathbf{Du} = 0$. We circumvent this by introducing an auxiliary variable $\mathbf{W} = \frac{\mathbf{Du}}{|\mathbf{Du}|}$, the equations for the Bingham flow are then reformulated as

$$\begin{cases} -\nabla\cdot(2\mu\mathbf{Du} + \tau_s\mathbf{W}) + \nabla p &= \mathbf{f}, \\ -\nabla\cdot\mathbf{u} &= 0, \\ \mathbf{W}|\mathbf{Du}| &= \mathbf{Du} \\ +B.C. & \end{cases}$$

In this talk we will address the discretization and linearization of these (nonlinear) equations. We will then propose a multilevel preconditioner with additive Schwartz smoothings for efficiently solving the resulting linear systems. Numerical experiments will be presented to demonstrate the effectiveness of both the nonlinear and linear solver.

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